Jour. Ind. Soc. Ag. Statistics
Vol. XLI, No. 3 (1989), pp. 283-287

A NOTE ON JIPS ESTIMATORS OF GHOSH AND GOMEZ

S. SAMPATH
Loyola College, Madras
(Received : February, 1988)

SUMMARY

In this paper the true bias and mean square error have been obtained for the JIPS estimators suggested by Ghosh and Gomez (1986) to the second degree approximation and some comments are given.

Keywords : Bias, Interpenetrating subsamples, Jackknife, Mean square error, Ratio estimation, simple random sample, variance.

Introduction

Consider a population of N units with y as the variable of interest and x as an auxiliary variable. Denote the population totals of the variables x and y over N population units by X and Y. The population ratio $R = Y/X$ is the unknown parameter of interest. We shall draw inference on R on the basis of k interpenetrating subsamples of size m each.

With notation as in Ghosh and Gomez [1] the two JIPS estimators proposed by them are

$$\hat{R}_3 = k\,\hat{R} - (k-1)\,\hat{R}_{1(.)}$$

$$\hat{R}_4 = m\,\hat{R}_2 - (m-1)\,\hat{R}_{2(.)}$$

The bias and mean square error of $\hat{R}_3$ and $\hat{R}_4$ are given in the following results.

***Result* : 1**

If the subsamples are drawn independently then for any new sampling design $\hat{R}_3$ is unbiased for R and B_4 is

$$[m\, B_4^{(2)}/k\, X^2] - [(m-1)\, B_4^{(1)}/k\, X^2]$$

for the second degree approximation.

PROOF :

For $u = 1,2,\ldots,k$; $\quad i = 1,2,\ldots,k,$

taking $\quad \hat{Y}_{(u)} = Y(1+e_{0u}),\ \hat{X}_{(u)} = X(1+e_{1u}),$

$$\hat{Y}_i = Y(1+e_{0i}),\ \hat{X}_i = X(1+e_{1i}),$$

write

$$B_3 = k\,R\left\{ E\left[\sum_{i=1}^{k} e_{1i}^2/k^2\right] - E\left[\sum_{i=1}^{k} e_{0i}\, e_{1i}/k^2\right]\right\}$$

$$-(k-1)\,R\left\{E\left[\sum_{u=1}^{k} e_1^2\ /k\right] - E\left[\sum_{u=1}^{k} e_{0u}\ e_{1u}/k\right]\right\}$$

Since,

$$E\left[\sum_{u=1}^{k} e_{1u}^2\right] = E\left[\sum_{i=1}^{k} e_{1i}^2/(k-1)\right]$$

and $\quad E\left[\sum_{u=1}^{k} e_{0u}\, e_{1u}\right] = E\left[\sum_{i=1}^{k} e_{0i}\, e_{1i}/\,(k-1)\right]$

we get $\quad B_3 = 0$

For $\quad j = 1,2,\ldots,k$; $\quad v = 1,2,\ldots,m,\quad$ taking

$$\hat{Y}_{j(v)} = Y[1+e_{0j(v)}],\quad \hat{X}_{j(v)} = X[1+e_{1j(v)}]$$

write

$$B_4 = m\,R\left\{E\left[\sum_{j=1}^{k} e_{1j}^2/k\right] - E\left[\sum_{j=1}^{k} e_{0j}\, e_{1j}/k\right]\right\}$$

$$-\,(m-1)\,R\left\{E\left[\sum_{v=1}^{m}\sum_{j=1}^{k} e_{1j(v)}^2/m\,R\,k\right]\right.$$

$$\left.-\,E\left[\sum_{v=1}^{m}\sum_{j=1}^{k} e_{1j(v)}\, e_{oj(v)}/m\,R\,k\right]\right\}$$

and the required result, is obtained.

Result : 2

To the second degree approximation the mean square error of $\hat{R}_3$ is $(2\,k^2 - 4\,k + 3)\,R^2\,\{[v\,(\hat{Y})/Y^2] + [v\,(\hat{X})/X^2] - 2\,[\mathrm{Cov}\,(\hat{X},\hat{Y})XY]\}$

Proof of the result is straightforward.

Results : 3

When the subsamples are drawn independently, under SRSWR and SRSWOR, to the second degree approximation, the mean square error of $\hat{R}_4$ is

$$R^2\,\{[v(\hat{Y})/Y^2] + [v\,(\hat{X})/X^2] - [2\,\mathrm{Cov}\,(\hat{X},\,\hat{Y})/XY]\}$$

PROOF :

In SRSWR and SRSWOR,

$$\hat{X}_j = N\left(\sum_{i=1}^{m} x_{ij}/m\right),\quad Y_i = N\left(\sum_{i=1}^{m} y_{ij}/m\right),$$

$$\hat{X}_{j(v)} = N\left[\sum_{\substack{i=1\\ \neq v}}^{m} x_{ij}/(m-1)\right],\ \hat{Y}_{i(v)} = N\left[\sum_{\substack{i=1\\ \neq v}}^{m} y_{ij}/(m-1)\right]$$

Since $\sum_{\nu=1}^{m} \hat{X}_{j(\nu)} = m \hat{X}_j$ and $\sum_{\nu=1}^{m} \hat{Y}_{j(\nu)} = m \hat{Y}_j$, we have

$$me_{oj} = \sum_{\nu=1}^{m} e_{oj(\nu)} \text{ and } me_{1j} = \sum_{\nu=1}^{m} e_{1j(\nu)}$$

Therefore $M(\hat{R}_4) = R^2 E\left[\sum_{j=1}^{k} (e_{0j}^2 + e_{1j}^2 - 2\, e_{oj}\, e_{1j})/k^2\right]$

which gives the required result

REMARKS

1. From Result 1, it is inferred that, when the subsamples are drawn independently using any sampling design, to the second degree approximation, $\hat{R}_3$ is always unbiased. Hence defining an estimate for B_3 as suggested by Ghosh and Gomez seems to be not necessary.

2. For B_3, Ghosh and Gomez have proposed the estimator $\hat{B}_3$ whose expected value is

$$(m-1)\,[B_4^{(1)} - B_4^{(2)}]/kX^2,$$

while the true bias is

$$[m\, B_4^{(2)} - (m-1)\; B_4^{(1)}]/kX^2$$

3. Since the mean square error of the conventional ratio estimator is

$$R^2\{[V(\hat{Y})/Y^2] + [V(\hat{X})/X^2] - [2 \operatorname{Cov}(\hat{X},\hat{Y})/X\,Y]\},$$

it is noted that from the Results 2 and 3, for $k \geq 1$, the mean square error of $\hat{R}_3$ is always larger than that of the conventional estimator and the mean square error of $\hat{R}_4$ is same as that of the conventional estimator when samples are drawn by either SRSWR or SRSWOR.

It is concluded that the estimators $\hat{R}_3$ and $\hat{R}_4$ are in no way superior to the usual ratio estimator, when simple random sampling is adopted,

ACKNOWLEDGEMENT

The author is grateful to Dr. K. Suresh Chandra. Department of Statistics, University of Madras for his help and guidance and is also grateful to the referee for many useful suggestions.

REFERENCE

[1] Ghosh, S. and Gomez. R. (1986): Comparision of Ratio estimators based on interpenetrating subsamples with or without jackknifing, *Jour. Ind. Soc. Ag. Statistics*, 38 : 200-210.